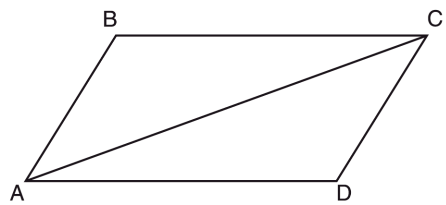
G.G.27: Quadrilateral Proofs: Write a proof arguing from a given hypothesis to a given conclusion

1 Given that *ABCD* is a parallelogram, a student wrote the proof below to show that a pair of its opposite angles are congruent.

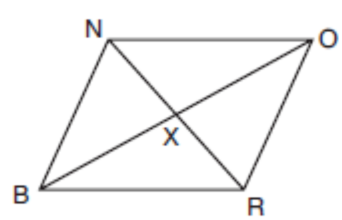


Statement	Reason
1. ABCD is a parallelogram.	1. Given
2. $\overline{BC} \cong \overline{AD}$ $\overline{AB} \cong \overline{DC}$	2. Opposite sides of a parallelogram are congruent.
3. $\overline{AC} \cong \overline{CA}$	3. Reflexive Postulate of Congruency
4. $\triangle ABC \cong \triangle CDA$	4. Side-Side-Side
5. $\angle B \cong \angle D$	5. ______________________

What is the reason justifying that $\angle B \cong \angle D$?

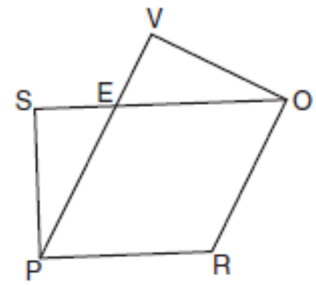
1) Opposite angles in a quadrilateral are congruent.
2) Parallel lines have congruent corresponding angles.
3) Corresponding parts of congruent triangles are congruent.
4) Alternate interior angles in congruent triangles are congruent.

2 The accompanying diagram shows quadrilateral *BRON*, with diagonals $\overline{NR}$ and $\overline{BO}$, which bisect each other at *X*.



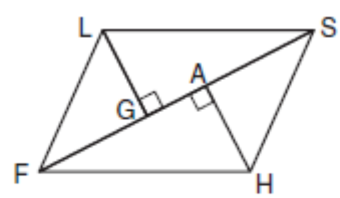
Prove: $\Delta BNX \cong \Delta ORX$

3 Given: *PROE* is a rhombus, $\overline{SEO}$, $\overline{PEV}$, $\angle SPR \cong \angle VOR$



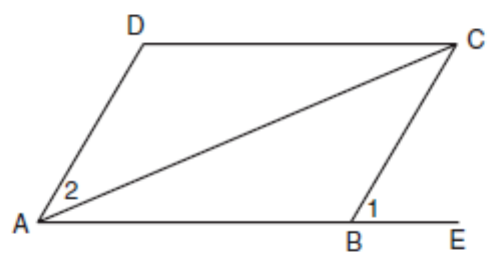
Prove: $\overline{SE} \cong \overline{EV}$

4 Given: parallelogram *FLSH*, diagonal $\overline{FGAS}$, $\overline{LG} \perp \overline{FS}$, $\overline{HA} \perp \overline{FS}$



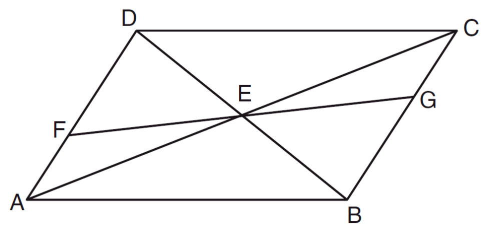
Prove: $\triangle LGS \cong \triangle HAF$

5 Given: parallelogram *ABCD*, diagonal $\overline{AC}$, and $\overline{ABE}$



Prove: $m\angle 1 > m\angle 2$

6 In the diagram below of quadrilateral $ABCD$, $\overline{AD} \cong \overline{BC}$ and $\angle DAE \cong \angle BCE$. Line segments AC, DB, and FG intersect at E.
Prove: $\triangle AEF \cong \triangle CEG$

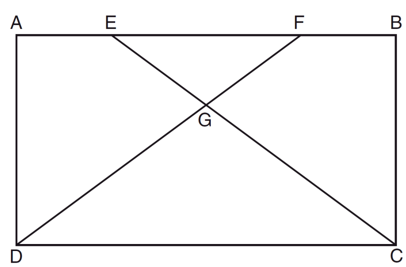


7 Given: Quadrilateral $ABCD$ with $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{BC}$, and diagonal $\overline{BD}$ is drawn.
Prove: $\angle BDC \cong \angle ABD$

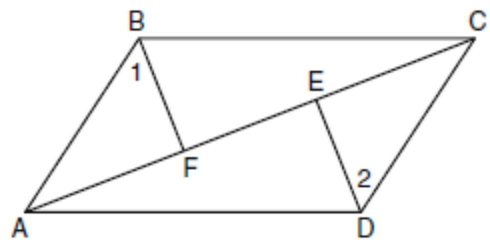
8 Prove that the diagonals of a parallelogram bisect each other.

9 A tricolored flag is made out of a rectangular piece of cloth whose corners are labeled A, B, C, and D. The colored regions are separated by two line segments, $\overline{BM}$ and $\overline{CM}$, that meet at point M, the midpoint of side $\overline{AD}$. Prove that the two line segments that separate the regions will always be equal in length, regardless of the size of the flag.

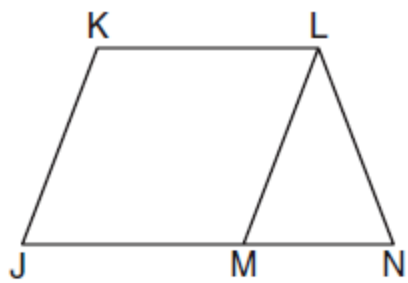
10 The diagram below shows rectangle $ABCD$ with points E and F on side $\overline{AB}$. Segments $\overline{CE}$ and $\overline{DF}$ intersect at G, and $\angle ADG \cong \angle BCG$. Prove: $\overline{AE} \cong \overline{BF}$



11 Given: Quadrilateral *ABCD*, diagonal $\overline{AFEC}$, $\overline{AE} \cong \overline{FC}$, $\overline{BF} \perp \overline{AC}$, $\overline{DE} \perp \overline{AC}$, $\angle\mathbf{1} \cong \angle\mathbf{2}$
Prove: *ABCD* is a parallelogram.



12 Given: *JKLM* is a parallelogram.
$\overline{JM} \cong \overline{LN}$
$\angle LMN \cong \angle LNM$
Prove: *JKLM* is a rhombus.



1 ANS: 3 REF: 081208ge

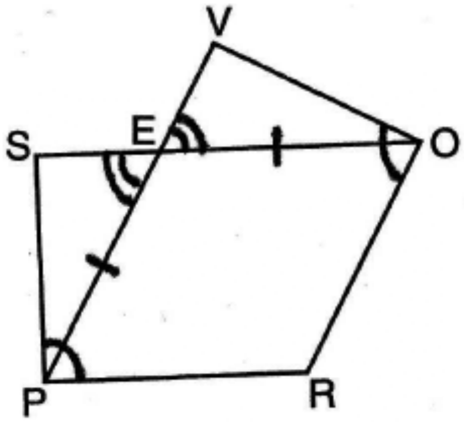
2 ANS:

Because diagonals $\overline{NR}$ and $\overline{BO}$ bisect each other, $\overline{NX} \cong \overline{RX}$ and $\overline{BX} \cong \overline{OX}$. $\angle BXN$ and $\angle OXR$ are congruent vertical angles. Therefore $\Delta BNX \cong \Delta ORX$ by SAS.

REF: 080731b

3 ANS:

Because $PROE$ is a rhombus, $\overline{PE} \cong \overline{OE}$. $\angle SEP \cong \angle VEO$ are congruent vertical angles. $\angle EPR \cong \angle EOR$ because opposite angles of a rhombus are congruent. $\angle SPE \cong \angle VOE$ because of the Angle Subtraction Theorem. $\Delta SEP \cong \Delta VEO$ because of ASA. $\overline{SE} \cong \overline{EV}$ because of CPCTC.



REF: 010934b

4 ANS:

Because $FLSH$ is a parallelogram, $\overline{FH} \cong \overline{SL}$. Because $FLSH$ is a parallelogram, $\overline{FH} \parallel \overline{SL}$ and since $\overline{FGAS}$ is a transversal, $\angle AFH$ and $\angle LSG$ are alternate interior angles and congruent. Therefore $\Delta LGS \cong \Delta HAF$ by AAS.

REF: 010634b

5 ANS:

Because $ABCD$ is a parallelogram, $\overline{AD} \parallel \overline{CB}$ and since $\overline{ABE}$ is a transversal, $\angle BAD$ and $\angle 1$ are corresponding angles and congruent. If $m\angle BAD > m\angle 2$, then $m\angle 1 > m\angle 2$, using substitution.

REF: 060533b

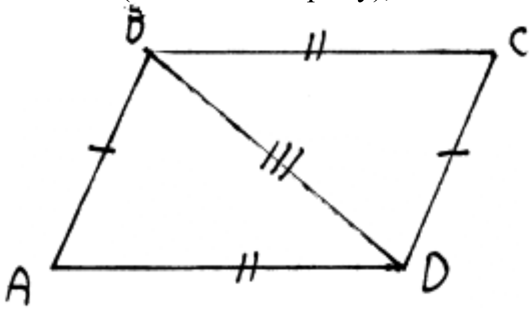
6 ANS:

Quadrilateral $ABCD$, $\overline{AD} \cong \overline{BC}$ and $\angle DAE \cong \angle BCE$ are given. $\overline{AD} \parallel \overline{BC}$ because if two lines are cut by a transversal so that a pair of alternate interior angles are congruent, the lines are parallel. $ABCD$ is a parallelogram because if one pair of opposite sides of a quadrilateral are both congruent and parallel, the quadrilateral is a parallelogram. $\overline{AE} \cong \overline{CE}$ because the diagonals of a parallelogram bisect each other. $\angle FEA \cong \angle GEC$ as vertical angles. $\Delta AEF \cong \Delta CEG$ by ASA.

REF: 011238ge

7 ANS:

$\overline{BD} \cong \overline{DB}$ (Reflexive Property); $\Delta ABD \cong \Delta CDB$ (SSS); $\angle BDC \cong \angle ABD$ (CPCTC).



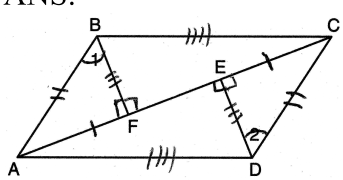
REF: 061035ge

8 ANS:
Assume parallelogram *JMAP* with diagonals intersecting at *O*. Opposite sides of a parallelogram are congruent, so $\overline{JM} \cong \overline{AP}$. $\angle JOM$ and $\angle AOP$ are congruent vertical angles. Because *JMAP* is a parallelogram, $\overline{JM} \parallel \overline{AP}$ and since $\overline{JOA}$ is a transversal, $\angle MJO$ and $\angle PAO$ are alternate interior angles and congruent. Therefore $\Delta MJO \cong \Delta PAO$ by AAS. Corresponding parts of congruent triangles are congruent. Therefore $\overline{JO} \cong \overline{AO}$ and $\overline{MO} \cong \overline{PO}$ and the diagonals of a parallelogram bisect each other.
REF: 010233b

9 ANS:
$\overline{AB} \cong \overline{CD}$, because opposite sides of a rectangle are congruent. $\overline{AM} \cong \overline{DM}$, because of the definition of midpoint. $\angle A$ and $\angle D$ are right angles because a rectangle has four right angles. $\angle A \cong \angle D$, because all right angles are congruent. $\Delta ABM \cong \Delta DCM$, because of SAS. $\overline{BM} \cong \overline{CM}$ because of CPCTC.
REF: 080834b

10 ANS:
Rectangle *ABCD* with points *E* and *F* on side $\overline{AB}$, segments $\overline{CE}$ and $\overline{DF}$ intersect at *G*, and $\angle ADG \cong \angle BCE$ are given. $\overline{AD} \cong \overline{BC}$ because opposite sides of a rectangle are congruent. $\angle A$ and $\angle B$ are right angles and congruent because all angles of a rectangle are right and congruent. $\Delta ADF \cong \Delta BCE$ by ASA. $\overline{AF} \cong \overline{BE}$ per CPCTC. $\overline{EF} \cong \overline{FE}$ under the Reflexive Property. $\overline{AF} - \overline{EF} \cong \overline{BE} - \overline{FE}$ using the Subtraction Property of Segments. $\overline{AE} \cong \overline{BF}$ because of the Definition of Segments.
REF: 011338ge

11 ANS:



$\overline{FE} \cong \overline{FE}$ (Reflexive Property); $\overline{AE} - \overline{FE} \cong \overline{FC} - \overline{EF}$ (Line Segment Subtraction Theorem); $\overline{AF} \cong \overline{CE}$ (Substitution); $\angle BFA \cong \angle DEC$ (All right angles are congruent); $\Delta BFA \cong \Delta DEC$ (AAS); $\overline{AB} \cong \overline{CD}$ and $\overline{BF} \cong \overline{DE}$ (CPCTC); $\angle BFC \cong \angle DEA$ (All right angles are congruent); $\Delta BFC \cong \Delta DEA$ (SAS); $\overline{AD} \cong \overline{CB}$ (CPCTC); *ABCD* is a parallelogram (opposite sides of quadrilateral *ABCD* are congruent) REF: 080938ge

12 ANS: $\overline{JK} \cong \overline{LM}$ because opposite sides of a parallelogram are congruent. $\overline{LM} \cong \overline{LN}$ because of the Isosceles Triangle Theorem. $\overline{LM} \cong \overline{JM}$ because of the transitive property. *JKLM* is a rhombus because all sides are congruent.
REF: 011036ge